Exercice 1

Calculer les intégrales suivantes :

1) $I_1=\int_0^4 x^4dx$; $I_2=\int_0^1(x^2+3x+2)dx$; $I_3=\int_1^2\frac{1}{x^4}dx$; $I_4=\int_1^2 x^{\frac{3}{2}}dx$; $I_5=\int_1^2\sqrt[4]{x^3}dx$; $I_6=\int_1^2 x^{-\frac{2}{3}}dx$.

2) $I_2=\int_0^1\frac{1}{x+1}dx$; $I_3=\int_0^1\frac{1}{2x+1}dx$; $I_4=\int_0^1\frac{x}{x^2+1}dx$; $I_5=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\cos x}{\sin x}dx$; $I_6=\int_0^1\frac{x^2}{x^3+2}dx$;

$I_7=\int_0^1\frac{e^x}{e^x+1}dx$; $I_8=\int_0^1\frac{x^2+1}{x^3+3x+1}dx$; $I_9=\int_{\sqrt{e}}^{e}\frac{1}{x\ln x}dx$.

3) $I_1=\int_1^2\frac{x+1}{x}dx$; $I_2=\int_0^1\frac{x^2+3x+2}{x+2}dx$; $I_3=\int_0^1\frac{2x-1}{x+1}dx$; $I_4=\int_0^1\frac{2}{(x+1)(x-3)}dx$;

$I_5=\int_0^1\frac{x}{(x-2)(x+2)}dx$; $I_6=\int_{-1}^0\frac{2x-1}{(x-1)(x+3)}dx$; $I_7=\int_0^1\frac{x+1}{x^2+5x+6}dx$; $I_8=\int_0^1\frac{3}{x^2+3x+2}dx$;

$I_9=\int_0^1\frac{x^2+x+1}{x^2+4x+3}dx$; $I_{10}=\int_1^2\frac{1-x}{x(x^2-9)}dx$.

4) $I_2=\int_0^1-\frac{2x}{(x^2+1)^2}dx$; $I_3=\int_0^1-\frac{3x^2+2x+1}{(x^3+x^2+x+2)^2}dx$; $I_4=\int_0^1\frac{2x+1}{(x^2+x+3)^2}dx$; $I_5=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\sin x}{\cos^2 x}dx$;

$I_6=\int_0^1\frac{2e^x}{(e^x+2)^2}dx$; $I_7=\int_{\sqrt{e}}^{e}\frac{1}{x(\ln x)^2}dx$; $I_8=\int_0^1\frac{1}{2(x+1)\sqrt{x+1}}dx$; $I_9=\int_1^2\frac{3}{\sqrt{x^3}}dx$.

5) $I_2=\int_0^1\frac{1}{2\sqrt{x+1}}dx$; $I_3=\int_0^1\frac{x}{\sqrt{x^2+1}}dx$; $I_4=\int_0^1\frac{3x}{\sqrt{2x^2+3}}dx$; $I_5=\int_0^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sqrt{1+\cos^2 x}}dx$;

$I_7=\int_1^e\frac{\ln x}{x\sqrt{1+\ln^2 x}}dx$; $I_8=\int_e^{e^2}\frac{1}{x\sqrt{2+\ln x}}dx$.

6) $I_1=\int_0^{\frac{\pi}{3}}\sin x(\cos x)^2dx$; $I_3=\int_0^1 x(x^2+1)^5dx$; $I_4=\int_1^e\frac{(\ln x)^4}{x}dx$; $I_5=\int_0^1 x\left(\sqrt{x^2+1}\right)^3dx$;

$I_6=\int_0^{\ln 2}e^{2x}(e^{2x}+1)^3dx$.

7) $I_1=\int_0^{\frac{\pi}{3}}\cos\left(2x+\frac{\pi}{3}\right)dx$; $I_2=\int_0^{\frac{\pi}{3}}\sin\left(3x-\frac{\pi}{3}\right)dx$; $I_3=\int_0^{\frac{\pi}{3}}\cos^3 xdx$; $I_4=\int_0^{\frac{\pi}{4}}\cos^4 xdx$;

$I_5=\int_0^{\frac{\pi}{2}}\cos^5 xdx$; $I_6=\int_0^{\frac{\pi}{3}}\sin^2 xdx$; $I_7=\int_0^{\frac{\pi}{2}}\sin^3 xdx$; $I_8=\int_0^{\frac{\pi}{2}}\sin^4 xdx$.

Exercice 2

En utilisant une intégration par parties, calculer les intégrales suivantes :

$I_1=\int_0^{\frac{\pi}{3}}x\cos xdx$; $I_2=\int_0^{\frac{\pi}{2}}x\sin xdx$; $I_3=\int_1^e x\ln xdx$; $I_4=\int_1^e\ln xdx$; $I_5=\int_1^e\frac{\ln x}{\sqrt{x}}dx$;

$I_6=\int_0^1 x\ln(x+1)dx$; $I_7=\int_0^{\frac{\pi}{3}}x^2\cos\left(4x+\frac{\pi}{3}\right)dx$; $I_8=\int_0^1\ln\left(x+\sqrt{x^2+1}\right)dx$; $I_9=\int_0^1(2x+3)\ln(x+1)dx$;

$I_{10}=\int_{\sqrt{e}}^{e}\frac{x\ln x}{(x^2+1)^2}dx$ $I_{11}=\int_0^1 xe^xdx$; $I_{12}=\int_0^1 xe^{2x-1}dx$; $I_{13}=\int_{-\ln 2}^0(x+1)e^{-x}dx$;

$I_{14}=\int_0^{\ln x}(x^2+1)e^{2x}dx$; $I_{15}=\int_{\frac{1}{2}}^1\frac{x}{\sqrt{1+2x}}dx$; $I_{16}=\int_{-\ln 2}^0 e^{-x}\ln(1+2e^x)dx$.

Exercice 3

Calculer les intégrales suivantes :

$$I_1=\int_0^1\frac{dx}{1+x^2}\ ;\ I_2=\int_0^1\frac{3}{2+2x+x^2}dx\ ;\ I_3=\int_0^1\frac{e^t}{1+e^{2t}}dt\ ;\ I_4=\int_1^e\frac{1}{x\left(1+\ln^2 x\right)}dx\ ;$$

$$I_5=\int_0^{\frac{\pi}{3}}\frac{\cos x}{1+\sin^2 x}dx\ ;\ I_6=\int_4^9\frac{1}{2\sqrt{x}(1+x)}dx\ ;\ I_7=\int_{-2}^{-4}\frac{7}{5+4x+x^2}dx\ ;\ I_8=\int_{-1}^0\frac{1}{2x^2+6x+5}dx$$

Exercice 4

Calculer les intégrales suivantes :

$$J_1=\int_0^5|x-2|dx\ ;\ J_2=\int_0^3|(x-1)(x+2)|dx\ ;\ J_3=\int_{-3}^{-1}\frac{|x+2|}{\left(x^2+4x\right)}dx\ ;\ J_4=\int_2^5\left|x^2-3x-4\right|dx\ ;$$

$$J_5=\int_0^{\sqrt{\ln 2}}xe^{-x^2}dx\ ;\ J_6=\int_{\ln 2}^{\ln 3}\frac{e^x-e^{-x}}{e^x+e^{-x}}dx\ ;\ J_7=\int_0^1\sqrt{3x+2}\,dx\ ;\ J_8=\int_1^e\frac{\ln^3 x}{x}dx\ ;$$

Exercice 5

En utilisant une intégration par changement de variable, calculer les intégrales suivantes :

$$K_1=\int_0^1\frac{\ln\left(1+e^t\right)}{1+e^{-t}}dt\ (Poser\ x=1+e^t)\ ;\ K_2=\int_1^2\frac{dx}{x+2\sqrt{x-1}}\ (Poser\ t=\sqrt{x-1})\ ;$$

$$K_3=\int_{\sqrt{3}}^{2\sqrt{2}}\frac{x}{\sqrt{x^2+1}-1}dx\ (Poser\ t=\sqrt{x^2+1})\ ;\ K_4=\int_0^{\ln 2}\frac{dx}{\sqrt{x^2+1}}\ (Poser\ x=\frac{e^t+e^{-t}}{2})\ ;$$

$$K_5=\int_{\sqrt{2}}^2\frac{dx}{x\sqrt{x^2-1}}\ (Poser\ t=\sqrt{x^2-1})\ ;\ K_6=\int_{\ln 2}^{\ln 4}\frac{dx}{\sqrt{e^x-1}}\ (Poser\ t=\sqrt{e^x-1})\ ;$$

$$K_7=\int_1^3\frac{dx}{(1+x)\sqrt{x}}\ (Poser\ t=\sqrt{x})\ ;\ K_8=\int_{e^{-2}}^e\frac{dt}{t\sqrt{3+\ln t}}\ (Poser\ x=\ln t)\ ;$$