Exercice 1

Calculer les intégrales suivantes :

1) $I_1 = \int_0^4 x^4 dx$; $I_2 = \int_0^1 (x^2+3x+2)dx$; $I_3 = \int_1^2 \frac{1}{x^4}dx$; $I_4 = \int_1^2 x^{\frac{3}{2}}dx$; $I_5 = \int_1^2 \sqrt[4]{x^3}dx$; $I_6 = \int_1^2 x^{-\frac{2}{3}}dx$.

2) $I_2 = \int_0^1 \frac{1}{x+1}dx$; $I_3 = \int_0^1 \frac{1}{2x+1}dx$; $I_4 = \int_0^1 \frac{x}{x^2+1}dx$; $I_5 = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos x}{\sin x}dx$; $I_6 = \int_0^1 \frac{x^2}{x^3+2}dx$;

$I_7 = \int_0^1 \frac{e^x}{e^x+1}dx$; $I_8 = \int_0^1 \frac{x^2+1}{x^3+3x+1}dx$; $I_9 = \int_{\sqrt{e}}^e \frac{1}{x\ln x}dx$.

3) $I_1 = \int_1^2 \frac{x+1}{x}dx$; $I_2 = \int_0^1 \frac{x^2+3x+2}{x+2}dx$; $I_3 = \int_0^1 \frac{2x-1}{x+1}dx$; $I_4 = \int_0^1 \frac{2}{(x+1)(x-3)}dx$;

$I_5 = \int_0^1 \frac{x}{(x-2)(x+2)}dx$; $I_6 = \int_{-1}^0 \frac{2x-1}{(x-1)(x+3)}dx$; $I_7 = \int_0^1 \frac{x+1}{x^2+5x+6}dx$; $I_8 = \int_0^1 \frac{3}{x^2+3x+2}dx$;

$I_9 = \int_0^1 \frac{x^2+x+1}{x^2+4x+3}dx$; $I_{10} = \int_1^2 \frac{1-x}{x(x^2-9)}dx$.

4) $I_2 = \int_0^1 -\frac{2x}{(x^2+1)^2}dx$; $I_3 = \int_0^1 -\frac{3x^2+2x+1}{(x^3+x^2+x+2)^2}dx$; $I_4 = \int_0^1 \frac{2x+1}{(x^2+x+3)^2}dx$; $I_5 = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x}dx$;

$I_6 = \int_0^1 \frac{2e^x}{(e^x+2)^2}dx$; $I_7 = \int_{\sqrt{e}}^e \frac{1}{x(\ln x)^2}dx$; $I_8 = \int_0^1 \frac{1}{2(x+1)\sqrt{x+1}}dx$; $I_9 = \int_1^2 \frac{3}{\sqrt{x^3}}dx$.

5) $I_2 = \int_0^1 \frac{1}{2\sqrt{x+1}}dx$; $I_3 = \int_0^1 \frac{x}{\sqrt{x^2+1}}dx$; $I_4 = \int_0^1 \frac{3x}{\sqrt{2x^2+3}}dx$; $I_5 = \int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\sqrt{1+\cos^2 x}}dx$;

$I_7 = \int_1^e \frac{\ln x}{x\sqrt{1+\ln^2 x}}dx$; $I_8 = \int_e^{e^2} \frac{1}{x\sqrt{2+\ln x}}dx$.

6) $I_1 = \int_0^{\frac{\pi}{3}} \sin x(\cos x)^2 dx$; $I_3 = \int_0^1 x(x^2+1)^5 dx$; $I_4 = \int_1^e \frac{(\ln x)^4}{x}dx$; $I_5 = \int_0^1 x(\sqrt{x^2+1})^3 dx$;

$I_6 = \int_0^{\ln 2} e^{2x}(e^{2x}+1)^3 dx$.

7) $I_1 = \int_0^{\frac{\pi}{3}} \cos\left(2x+\frac{\pi}{3}\right)dx$; $I_2 = \int_0^{\frac{\pi}{3}} \sin\left(3x-\frac{\pi}{3}\right)dx$; $I_3 = \int_0^{\frac{\pi}{3}} \cos^3 x dx$; $I_4 = \int_0^{\frac{\pi}{4}} \cos^4 x dx$;

$I_5 = \int_0^{\frac{\pi}{2}} \cos^5 x dx$; $I_6 = \int_0^{\frac{\pi}{3}} \sin^2 x dx$; $I_7 = \int_0^{\frac{\pi}{2}} \sin^3 x dx$; $I_8 = \int_0^{\frac{\pi}{2}} \sin^4 x dx$.

Exercice 2

En utilisant une intégration par parties, calculer les intégrales suivantes :

$I_1 = \int_0^{\frac{\pi}{3}} x\cos x dx$; $I_2 = \int_0^{\frac{\pi}{2}} x\sin x dx$; $I_3 = \int_1^e x\ln x dx$; $I_4 = \int_1^e \ln x dx$; $I_5 = \int_1^e \frac{\ln x}{\sqrt{x}}dx$;

$I_6 = \int_0^1 x\ln(x+1)dx$; $I_7 = \int_0^{\frac{\pi}{3}} x^2\cos\left(4x+\frac{\pi}{3}\right)dx$; $I_8 = \int_0^1 \ln\left(x+\sqrt{x^2+1}\right)dx$; $I_9 = \int_0^1 (2x+3)\ln(x+1)dx$;

$I_{10} = \int_{\sqrt{e}}^e \frac{x\ln x}{(x^2+1)^2}dx$ $I_{11} = \int_0^1 xe^x dx$; $I_{12} = \int_0^1 xe^{2x-1}dx$; $I_{13} = \int_{-\ln 2}^0 (x+1)e^{-x}dx$;

$I_{14} = \int_0^{\ln x} (x^2+1)e^{2x}dx$; $I_{15} = \int_{\frac{1}{2}}^1 \frac{x}{\sqrt{1+2x}}dx$; $I_{16} = \int_{-\ln 2}^0 e^{-x}\ln(1+2e^x)dx$.

Exercice 3

Calculer les intégrales suivantes :

$$J_1 = \int_0^5 |x-2|\,dx \;\; ; \;\; J_2 = \int_0^3 |(x-1)(x+2)|\,dx \;\; ; \;\; J_3 = \int_{-3}^{-1} \frac{|x+2|}{(x^2+4x)}\,dx \;\; ; \;\; J_4 = \int_2^5 |x^2-3x-4|\,dx \;\; ;$$

$$J_5 = \int_0^{\sqrt{\ln 2}} x e^{-x^2}\,dx \;\; ; \;\; J_6 = \int_{\ln 2}^{\ln 3} \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx \;\; ; \;\; J_7 = \int_0^1 \sqrt{3x+2}\,dx \;\; ; \;\; J_8 = \int_1^e \frac{\ln^3 x}{x}\,dx \;\; ;$$