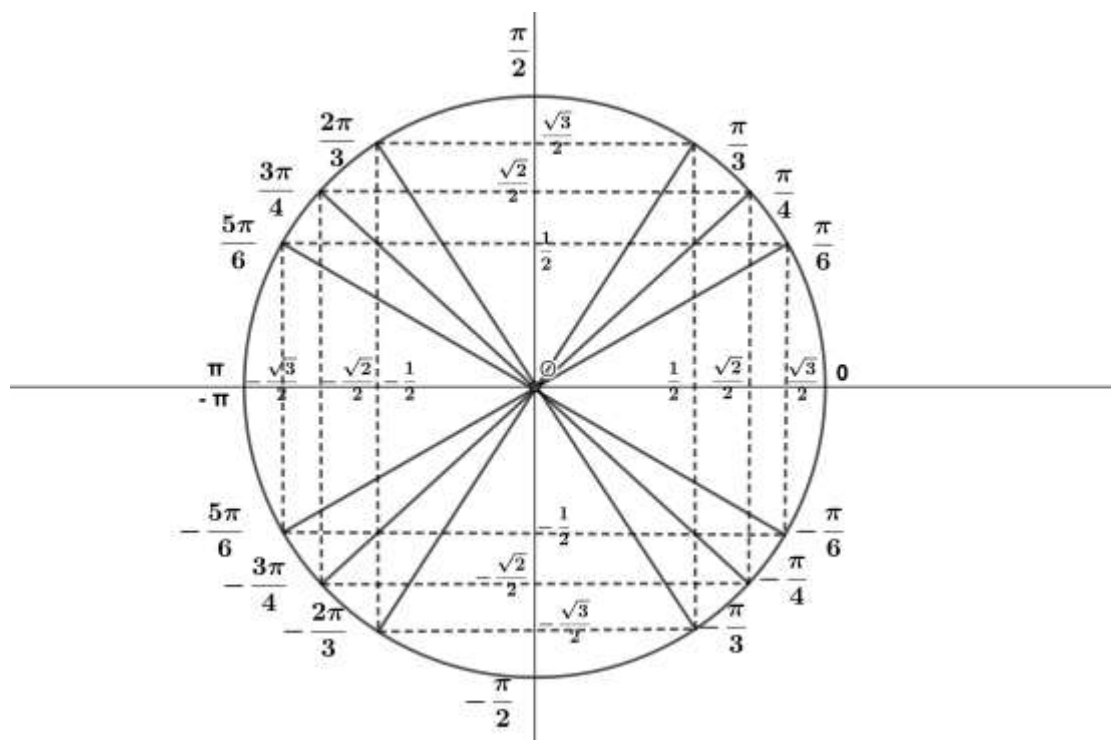




1 – Cercle trigonométrique



Cercle trigonométrique

2 – tableau des rapports trigonométriques usuels

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	\otimes	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

3 - Rappels

Propriétés

- ❖ $(\forall x \in \mathbb{R}), \cos^2 x + \sin^2 x = 1$
- ❖ $\left(\forall x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi / k \in \mathbb{Z} \right\} \right), \tan x = \frac{\sin x}{\cos x}$
- ❖ $\left(\forall x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi / k \in \mathbb{Z} \right\} \right), \frac{1}{\cos^2 x} = 1 + \tan^2 x$

Formules de transformations

$\cos(x + 2k\pi) = \cos x$	$\sin(x + 2k\pi) = \sin x$	$\tan(x + k\pi) = \tan x$
$\cos(-x) = \cos x$	$\sin(-x) = -\sin x$	$\tan(-x) = -\tan x$
$\cos(\pi - x) = -\cos x$	$\sin(\pi - x) = \sin x$	$\tan(\pi - x) = -\tan x$



$\cos(\pi + x) = -\cos x$	$\sin(\pi + x) = \sin x$	$\tan(\pi + x) = \tan x$
$\cos\left(\frac{\pi}{2} - x\right) = \sin x$	$\sin\left(\frac{\pi}{2} - x\right) = \cos x$	$\tan\left(\frac{\pi}{2} - x\right) = \frac{1}{\tan x}$
$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$	$\sin\left(\frac{\pi}{2} + x\right) = \cos x$	$\tan\left(\frac{\pi}{2} + x\right) = -\frac{1}{\tan x}$

4 - Equations trigonométriques

Proposition

Soit $\alpha \in \mathbb{R}$.

$$\begin{aligned} \diamond \cos x = \cos \alpha &\Leftrightarrow \begin{cases} x = \alpha + 2k\pi \\ \text{ou} \\ x = -\alpha + 2k\pi \end{cases} \quad (k \in \mathbb{Z}) \\ \diamond \sin x = \sin \alpha &\Leftrightarrow \begin{cases} x = \alpha + 2k\pi \\ \text{ou} \\ x = \pi - \alpha + 2k\pi \end{cases} \quad (k \in \mathbb{Z}) \\ \diamond \tan x = \tan \alpha &\Leftrightarrow x = \alpha + k\pi \quad (k \in \mathbb{Z}) \quad (\alpha \neq \frac{\pi}{2} + k'\pi, k' \in \mathbb{Z}) \end{aligned}$$

Cas particuliers

$\cos x = 0 \Leftrightarrow x \equiv \frac{\pi}{2} [\pi]$	$\sin x = 0 \Leftrightarrow x \equiv 0 [\pi]$	$\tan x = 0 \Leftrightarrow x \equiv 0 [\pi]$
$\cos x = 1 \Leftrightarrow x \equiv 0 [2\pi]$	$\sin x = 1 \Leftrightarrow x \equiv \frac{\pi}{2} [2\pi]$	$\tan x = 1 \Leftrightarrow x \equiv \frac{\pi}{4} [\pi]$
$\cos x = -1 \Leftrightarrow x \equiv \pi [2\pi]$	$\sin x = -1 \Leftrightarrow x \equiv -\frac{\pi}{2} [2\pi]$	$\tan x = -1 \Leftrightarrow x \equiv -\frac{\pi}{4} [\pi]$

5 - Formules de transformations

$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$	$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$	$\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b)$
$\tan(a + b) = \frac{\tan(a) + \tan(b)}{1 - \tan(a)\tan(b)}$	
$\tan(a - b) = \frac{\tan(a) - \tan(b)}{1 + \tan(a)\tan(b)}$	

6 - Formules de duplication

$\begin{aligned} \cos(2a) &= \cos^2(a) - \sin^2(a) \\ &= 2\cos^2(a) - 1 \\ &= 1 - 2\sin^2(a) \end{aligned}$	$\sin(2a) = 2\sin(a)\cos(a)$	$\tan(2a) = \frac{2\tan(a)}{1 - \tan^2(a)}$
$\cos(2a) = \frac{1 - \tan^2(a)}{1 + \tan^2(a)}$	$\sin(2a) = \frac{2\tan(a)}{1 + \tan^2(a)}$	

7 - Formules de linéarisation



$$\cos^2(a) = \frac{1 + \cos(2a)}{2}$$

$$\sin^2(a) = \frac{1 - \cos(2a)}{2}$$

$$\tan^2(a) = \frac{1 - \cos(2a)}{1 + \cos(2a)}$$

8 - Formules trigonométriques transformant le produit en somme

$$\cos(a)\cos(b) = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$

$$\sin(a)\sin(b) = -\frac{1}{2}[\cos(a+b) - \cos(a-b)]$$

$$\cos(a)\sin(b) = \frac{1}{2}[\sin(a+b) - \sin(a-b)]$$

$$\sin(a)\cos(b) = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

9 - Formules trigonométriques transformant une somme en un produit

$$\cos(p) + \cos(q) = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$$\cos(p) - \cos(q) = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$

$$\sin(p) + \sin(q) = 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$$\sin(p) - \sin(q) = 2\cos\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$

10 - Formule de transformation de $a\cos(x) + b\sin(x)$

Proposition

Soient a et b deux réels tels que $(a, b) \neq (0, 0)$. Alors :

$$\diamond (\exists \alpha \in \mathbb{R})(\forall x \in \mathbb{R}) : a\cos(x) + b\sin(x) = \sqrt{a^2 + b^2} \cos(x - \alpha)$$

$$\text{où } \cos \alpha = \frac{a}{\sqrt{a^2 + b^2}} \text{ et } \sin \alpha = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\diamond (\exists \beta \in \mathbb{R})(\forall x \in \mathbb{R}) : a\cos(x) + b\sin(x) = \sqrt{a^2 + b^2} \cos(x + \beta)$$

$$\text{Où } \cos \alpha = \frac{b}{\sqrt{a^2 + b^2}} \text{ et } \sin \alpha = \frac{a}{\sqrt{a^2 + b^2}}$$